Maggie Wen liu

1. Asset or nothing call means that when price is bigger than strike price, you get payoff the asset price, or you get nothing. So the price of asset or nothing call = E(st) = Se^(-qt)N(d1) , where

Then put s0=2000, r=0.5%, q=2%,=0.3, T=1/12, K1=1900, K2=2200 into the above formula.

When strike price is 1900, the price of asset or nothing call is **1463.06**

When strike price is 2200, the price of asset or nothing call is **283.391**

2 Monte Carlo

.When K = 1900:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sample size | estimate | Abs error | Std error | Time |
| **5,000** | **1500.19** | **37.1339** | **13.2745** | **0.001288 second** |
| 50,000 | **1463.67** | **0.608158** | **4.27616** | **0.013803 second** |
| 500,000 | **1462.28** | **0.775788** | **1.35237** | **0.127371 second** |
| 2,000,000 | **1462.04** | **1.01663** | **0.676248** | **0.53096 second** |
| 7,000,000 | **1462.22** | **0.844391** | **0.361486** | **1.79915 second** |
| 10,000,000 | **1462.43** | **0.633256** | **0.302407** | **2.56998second** |

When K = 2200:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sample size | estimate | Abs error | Std error | Time |
| **5,000** | **284.159** | **0.767795** | **10.6998** | **0.001216second** |
| 50,000 | **283.911** | **0.52007** | **3.38378** | **0.011768 second** |
| 500,000 | **282.117** | **1.27459** | **1.06725** | **0.120217 second** |
| 2,000,000 | **282.507** | **0.884282** | **0.533908** | **0.495421second** |
| 7,000,000 | **283.323** | **0.0677555** | **0.285729** | **1.74138 second** |
| 10,000,000 | **283.365** | **0.0265085** | **0.239069** | **2.48478 second** |

3. Control Variate

When K = 1900:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sample size | estimate | Abs error | Std error | Time |
| **5,000** | **1461.7** | **1.36248** | **8.05555** | **0.001892 second** |
| 50,000 | **1460** | **3.06452** | **2.53667** | **0.019279 second** |
| 500,000 | **1463.44** | **0.379179** | **0.190088** | **0.220025 second** |
| 2,000,000 | **1463.8** | **0.740108** | **0.40121** | **0.785889 second** |
| 7,000,000 | **1462.91** | **0.146037** | **0.214565** | **2.63117 second** |
| 10,000,000 | **1463.31** | **0.248573** | **0.179511** | **4.00503 second** |

When K = 2200:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sample size | estimate | Abs error | Std error | Time |
| **5,000** | **289.713** | **6.32195** | **8.05472** | **0.001769 second** |
| 50,000 | **282.774** | **0.617356** | **2.55912** | **0.018233 second** |
| 500,000 | **283.237** | **0.153738** | **0.806584** | **0.182282 second** |
| 2,000,000 | **282.741** | **0.649736** | **0.402925** | **0.750818 second** |
| 7,000,000 | **283.646** | **0.25524** | **0.215502** | **2.65832 second** |
| 10,000,000 | **283.364** | **0.0275554** | **0.180261** | **3.93008 second** |

The control variates for the k1 works better, as we can see that standard error generated by asset 1 is a little bit smaller than asset 2. Because strike price 1900 is smaller than 2200, the covariance between stock price and call price is higher and the correlation is higher. so according to , the standard error is lower, the better variance reduction

4. Importance sampling

When K = 1900:

Best \_hat is 0.028, the smallest standard error exists

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sample size | estimate | Abs error | Std error | Time |
| **5,000** | **1454** | **9.06145** | **12.1169** | **0.001383 second** |
| 50,000 | **1474.61** | **17.9842** | **9.49592** | **0.002571 second** |
| 500,000 | **1462.72** | **0.33851** | **1.201** | **0.159986 second** |
| 2,000,000 | **1463.28** | **0.224298** | **0.59966** | **0.589182 second** |
| 7,000,000 | **1463.09** | **0.0323598** | **0.320736** | **2.04951 second** |
| 10,000,000 | **1462.37** | **0.685814** | **0.268325** | **2.78852 second** |

When K = 2200: Best \_hat is 0.123, the smallest standard error exists

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sample size | estimate | Abs error | Std error | Time |
| **5,000** | **277.21** | **6.18091** | **4.605** | **0.001377 second** |
| 50,000 | **280.028** | **3.36294** | **1.4747** | **0.015731 second** |
| 500,000 | **282.398** | **0.993705** | **0.467939** | **0.136016 second** |
| 2,000,000 | **283.558** | **0.16703** | **0.234504** | **0.549788 second** |
| 7,000,000 | **283.548** | **0.157248** | **0.125406** | **1.9308 second** |
| 10,000,000 | **283.217** | **0.174596** | **0.104826** | **2.72583 second** |

Importance sampling is better for k=2200 due to the smaller standard error. The bigger strike price shit the center of g(x) to the right, so it is more accurate. Then, because for larger K, variance : Var(h(x)f(x)/g(x)) can reduce more. And for bigger K, exercising option is less likely to exercise and so the generation result is more precise.

5. For the asset or nothing call with the strike price 1900, control variates is the most efficient due to the smallest standard error. For the asset or nothing call with the strike price 2200 ,importance sampling is the most efficient due to the smallest standard error